Sydney
Technical
High School

Name: .... **Mathematics Class:** ....

Year 12

Mathematics Extension 1

HSC Course

Assessment 4 - Trial

August 2019

Time allowed: 120 minutes + 5 minutes reading time

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided

Section 1 Multiple Choice

Questions 1-10

10 Marks

Section II Questions 11-14

60 Marks

Section 1

Multiple Choice (10 marks)

Attempt Questions 1 - 10

Use the multiple-choice answer sheet for Questions 1-10

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1. What is the value of $\lim_{x \to 0} \dfrac{\sin\left(\frac{1}{2}x\right)}{2x}$?

A. 0

B. $\frac{1}{4}$

C. 1

D. 4

2. What are the coordinates of the point P that divides the interval joining the points $A(1,2)$ and $B(7,5)$ internally in the ratio 2:1?

A. (3,3)

B. (3,4)

C. (5,4)

D. (5,3)

3. When the polynomial $P(x) = x^3 - 5x^2 + kx + 2$ is divided by $(x + 1)$ the remainder is 3. What is the value of k?

A. -7

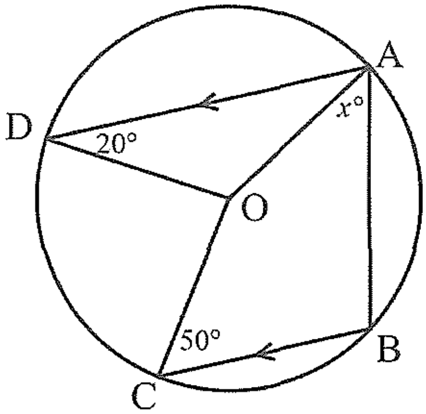
B. -5

C. 5

D. 7

4. A, B, C and D are points on a circle with centre O. BC is parallel to AD.

$\angle ADO = 20°$ and $\angle BCO = 50°$. Let $\angle BAO = x°$.



Not to scale

What is the value of x?

A. 15

B. 35

C. 40

D. 55

5. Which of the following is a simplification of $4\log_e\left(\sqrt{e^x}\right)$?

A. $4\sqrt{x}$

B. $\frac{1}{2}x$

C. $2x$

D. x^2

6. The acute angle between the lines $2x - y = 0$ and $kx - y = 0$ is equal to $\frac{\pi}{4}$.

What is the value of k?

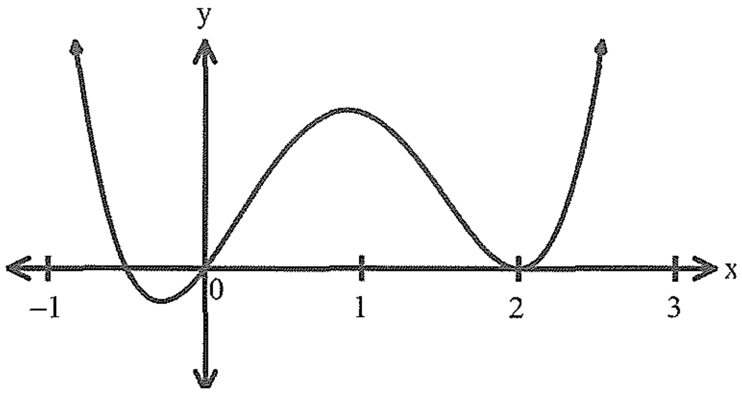
A. $k = -3$ or $k = -\frac{1}{3}$

B. $k = -3$ or $k = \frac{1}{3}$

C. $k = 3$ of $k = -\frac{1}{3}$

D. $k = 3$ or $k = \frac{1}{3}$

7. The graph of a polynomial function is shown below.



What could be the equation of the polynomial represented by the graph?

A. $y = x(2x - 1)(x - 1)^2$

B. $y = x(2x - 1)(x - 2)^2$

C. $y = x(2x + 1)(x - 2)^2$

D. $y = x(2x + 1)(x + 1)^2$

8. Which one of following is an expression for $\int \frac{3}{\sqrt{1 - 16x^2}}\, dx$?

A. $3\sin^{-1}(4x) + C$

B. $3\cos^{-1}(4x) + C$

C. $\frac{3}{4}\sin^{-1}(4x) + C$

D. $\frac{3}{4}\cos^{-1}(4x) + C$

9. Which is an expression for $\frac{d}{dx}(\tan^{-1}(2x + 1))$?

A. $\dfrac{1}{4x^2 + 4x + 2}$

B. $\dfrac{1}{2x^2 + 2x + 1}$

C. $\dfrac{1}{4x^2 + 2}$

D. $\dfrac{1}{2x^2 + 1}$

10. What is the general solution to the equation $2\sin^2\theta + 5\cos\theta + 1 = 0$

A. $2n\pi \pm \frac{2\pi}{3}$ where n is an integer.

B. $2n\pi \pm \frac{5\pi}{6}$ where n is an integer.

C. $2n\pi \pm \frac{\pi}{6}$ where n is an integer.

D. $2n\pi \pm \frac{\pi}{3}$ where n is an integer.

Section II

Total Marks (60)
Attempt Questions 11 – 14.

Answer each question in your writing booklet.

In Questions 11 - 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 mark)

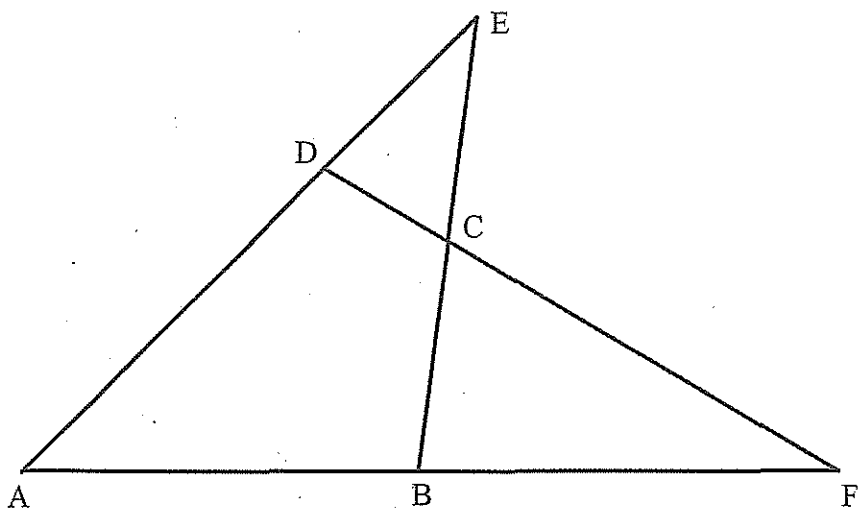
(a) Solve $\dfrac{3}{4x-1} \geq 2$ **3**

(b) Solve the equation $\sin 2x + \cos x = 0$ for $0 \leq x \leq 2\pi$ **3**

(c) Find the exact value of $\sin\left[\cos^{-1}\left(\dfrac{2}{3}\right) + \tan^{-1}\left(-\dfrac{3}{4}\right)\right]$. **3**

(d) Differentiate $x^2 \cos^{-1}(2x)$. **2**

(e) In the diagram $ABCD$ is a cyclic quadrilateral. AD produced and BC produced meet at E. AB produced and DC produced meet at F. $\angle DEC = \angle BFC$.



(i) Copy the diagram into your answer booklet.

(ii) Show that $\angle ADC = \angle ABC$. **2**

(ii) Show that AC is a diameter of the circle through A,B,C and D **2**

End of Question 11

Question 12 (15 mark) (start a new page)

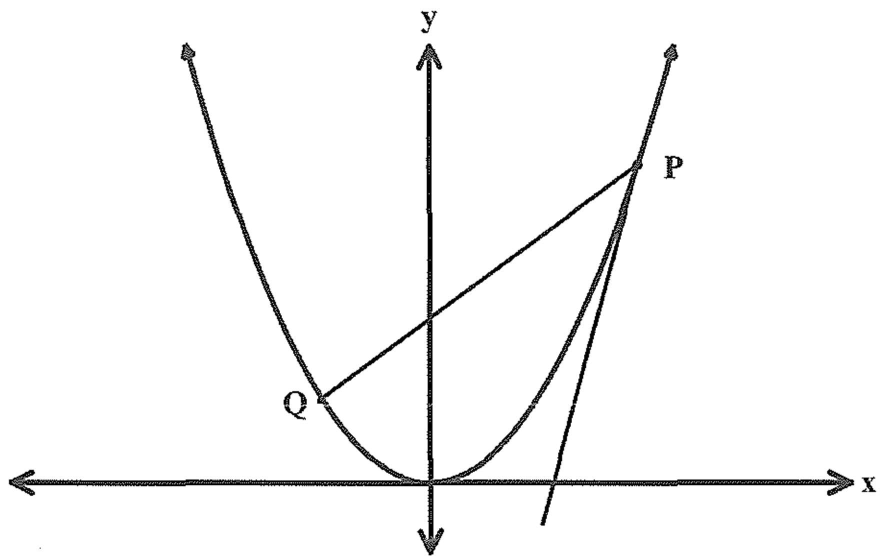
(a) Use Mathematical Induction to show that for all positive integers $n \geq 1$ **3**

$$1 \times 2^0 + 2 \times 2^1 + 3 \times 2^2 + \ldots + n \times 2^{n-1} = 1 + (n-1)\,2^n.$$

(b) An oil slick in the shape of a circle is spreading across a lake, such that its radius is increasing at a rate of 0.1 m/s.

Find the radius of the oil slick when its area is increasing at a rate of $2\pi m^2/s$. **2**

(c)



The diagram above shows the variable points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ on the parabola $x^2 = 4ay$.

M is the midpoint of PQ.

P and Q move such that the gradient of the tangent at P, is four times the gradient of the chord PQ.

(i) Show that $q = -\frac{1}{2}p$ **2**

(ii) Show that as p varies, M moves on a parabola. **2**

(d) Use the substitution $u = x + 1$ to evaluate in simplest exact form $\int_{2}^{5} \frac{x+2}{(x+1)^2}\,dx$. **3**

(e) The region bounded by the curve $y = \cos^{-1} x$ and the y axis between $y = \frac{\pi}{12}$ and $y = \frac{\pi}{4}$ is rotated through one complete revolution about the y-axis. Find the exact volume of the solid formed. **3**

End of Question 12

Question 13 (15 mark) (start a new page)

(a) At time t years the number N of individuals in a population is such that $N = P + 100e^{kt}$ for some constants P and k.

(i) Show that $\dfrac{dN}{dt} = k(N - P)$. **1**

(ii) If initially the population size is 600 and is increasing at a rate of 12 individuals per year, find the values of P and k. **2**

(b) On a certain day, low tide for a harbour occurs at 4:00 am and high tide occurs at 10:20 am. The corresponding depths are 3 m and 5 m respectively. The tidal motion is assumed to be simple harmonic.

(i) Show that the water depth, y metres, is given by $y = 4 - \cos\dfrac{3\pi t}{19}$ where t is the number of hours after low tide. **2**

(ii) A boat requires a depth of at least 4.5m. **2**
What is the earliest and the latest time that the boat can be in the harbour before 5:00pm on that day?

(c) A particle is moving in a straight line. At time t seconds it has displacement x metres to the right of a fixed point O on the line and velocity $v\ ms^{-1}$ given by $v = \dfrac{16 - x^2}{x}$.
Initially the particle is 1 metre to the right of O.

(i) Show that $x = \sqrt{16 - 15e^{-2t}}$. **3**

(ii) Find the limiting position of the particle. **1**

(d) Consider the function $f(x) = \sin^{-1}(1 - x) + \dfrac{\pi}{2}$.

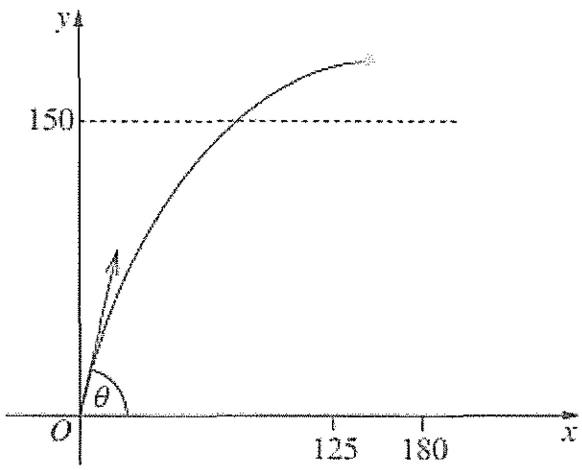
(i) Find the domain and range of the function. **2**

(ii) Sketch the graph of the function, clearly showing the shape of the curve and the co-ordinates of the endpoints. **2**

End of Question 13

Question 14 (15 mark) (start a new page)

(a) A firework is fired from O, on level ground, with velocity 70 metres per second at an angle of inclination θ. The equations of motion of the firework are $x = 70t\cos\theta$ and $y = 70t\sin\theta - 4.9t^2$. . (Do NOT prove this.)
The firework explodes when it reaches its maximum height.



(i) Show that the firework explodes at a height of $250\sin^2\theta$ metres **2**

(ii) Show that the firework explodes at a horizontal distance of $250\sin 2\theta$ metres from O. **1**

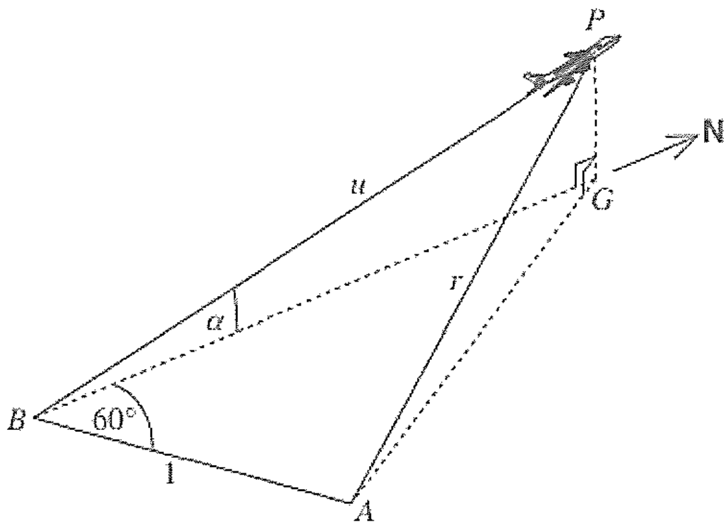
(iii) For best viewing, the firework must explode at a horizontal distance between 125 m and 180 m from O, and at least 150 m above the ground. For what values of θ will this occur? **3**

(b) (i) Show that $1 + x + x^2 + x^3 + ... + x^n = \dfrac{x^{n+1} - 1}{x - 1}$ **1**

(ii) Use the result from b(i) to show that

$$1 \times 1 + 2 \times 2 + 3 \times 4 + 4 \times 8 + ... + n \times 2^{n-1} = 2^n(n-1) + 1$$ **3**

(c) A plane P takes off from a point B. it flies due north at a constant angle α to the horizontal. An observer is located at A, 1 km from B, at a bearing 060° from B. Let u km be the distance from B to the plane and let r km be the distance from the observer to the plane. The point G is on the ground directly below the plane.



(i) Show that $r = \sqrt{1 + u^2 - u\cos\alpha}$. **3**

(ii) The plane is travelling at a constant speed of 360 km/h. At what rate, in terms of α, is the distance of the plane from the observer changing 5 minutes after take-off? **2**

End of Examination

☺☺☺

Question 14:

a) Please tell the examiner WHY you are setting $\dot{y} = 0$

a) (iii) Most students used their information found in parts (i) and (ii) to do this part.

If so, then one set of solutions gives that $\theta \geq 51^o$. The other set of solutions sees the solution to

$\frac{1}{2} \leq \sin 2\theta \leq \frac{18}{25}$. There are 2 sets of answers here: the first is in the first quadrant, and does not give angles greater than 51^o so offers no solution. Most students somehow got to this point. BUT there are a second set of solutions in quadrant 2, leading to $67^o \leq \theta \leq 75^o$. Not many were perceptive enough to get these.

The other "method" was to find the equation of the trajectory, then substitute in y = 150 and x = 125 then x = 180, and solving for Θ. This was very involved and led to many errors. Probably two students who did this 2 pages of work involved got anywhere.

b) (i) The formula is given. You cannot just "requote it"! The important part is that there are n+1 terms.

There were 3 methods:

1. The one in your solutions.
2. Multiply $(1 + x + x^2 + x^3 + \cdots)$ by $x - 1$
3. Let $S_n = 1 + x + x^2 + x^3 + \cdots. + x^n$

then $\quad x.S_n = x + x^2 + x^3 + \cdots + x^n + x^{n+1}$

So $\quad S_n\,(x-1) = x^{n+1} - 1$

$$\rightarrow S_n = 1 + x + x^2 + x^3 + \cdots. + x^n = \frac{x^{n+1}-1}{x-1}$$

For 1 mark, don't start using Mathematical Induction!

(b) (ii) Using Mathematical Induction was NOT following instructions to use part (b) (i) above., but would get you 2 marks out of 3 if it was done well (not often!)

(c) Last question, so meant to be difficult.

Year 12 Extension 1 Trial - 2019

Section 1

1	B	6.	B
2	C	7	C
3	A	8	C
4	D	9.	B
5	C	10.	A

1. $\lim_{x \to 0} \frac{\sin(\frac{1}{2}x)}{2x}$

$= \frac{1}{4} \lim_{x \to 0} \frac{\sin(\frac{1}{2}x)}{\frac{1}{2}x}$

$= \frac{1}{4}$ B

2. $(1,2) \quad (7,5) \quad 2:1$

$x = \frac{mx_2 + nx_1}{m+n}$ $\quad y = \frac{my_2 + ny_1}{m+n}$

$x = \frac{2\times 7 + 1\times 1}{3}$ $\quad y = \frac{2\times 5 + 1\times 2}{3}$

$x = 5$ $\quad y = 4$ C

3. $P(-1) = 3$

$x^3 - 5x^2 + kx + 2 = (-1)^3 - 5(-1) + k(-1) + 2$

$3 = 1 - 5 - k + 2$

$3 = -4 - k$

$k = -7$ A

4. Construct OF || AD

Join O to B

$\angle DOC = 20^\circ + 50^\circ = 70^\circ$

$\angle DOA = 140^\circ$

$\angle COB = 80^\circ$

$\angle BOA = 70^\circ$

$2x + 70^\circ = 180^\circ$

$x = 55^\circ$ D

5. $4 \log_e \sqrt{e^x}$

$= 4 \log_e e^{\frac{x}{2}}$

$= 4 \times \frac{1}{2} \log_e e^x$

$= 4 \times \frac{1}{2} x$

$= 2x$ C

6. $\tan \frac{\pi}{4} = 1$

$1 = \left| \frac{k-2}{1+2k} \right|$

$1+2k = k-2$ or $1+2k = -(k-2)$

$k = -3$ $3k = 1$

$k = \frac{1}{3}$ B

7. $(x-2)^2$ double root

x single root

root in between -1 and 0. C

8 $$\int \frac{3}{\sqrt{1-16x^2}}\,dx = 3\int \frac{1}{\sqrt{1-16x^2}}\,dx$$

$$= 3\int \frac{1}{\sqrt{16(\frac{1}{16}-x^2)}}\,dx$$

$$= \frac{3}{4}\sin^{-1}(4x) + C$$ C

9. $$\frac{d}{dx}\tan^{-1}(2x+1) = \frac{2}{1+(2x+1)^2}$$

$$= \frac{2}{4x^2+4x+2}$$

$$= \frac{1}{2x^2+2x+1}$$ B

10 $$2\sin^2\theta + 5\cos\theta + 1 = 0$$

$$2(1-\cos^2\theta) + 5\cos\theta + 1 = 0$$

$$2 - 2\cos^2\theta + 5\cos\theta + 1 = 0$$

$$2\cos^2\theta - 5\cos\theta - 3 = 0$$

$$(2\cos\theta + 1)(\cos\theta - 3) = 0$$

$$\cos\theta = -\tfrac{1}{2}$$ reject

$$\theta = 2n\pi \pm \cos^{-1}(-\tfrac{1}{2})$$

$$= 2n\pi \pm \frac{2\pi}{3}$$ A

Question 11

a) $\dfrac{3}{4x-1} \geq 2$

$3(4x-1) \geq 2(4x-1)^2$

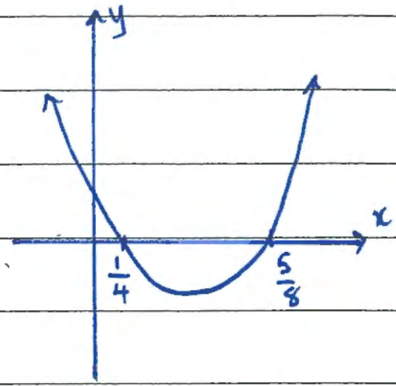
$0 \geq 2(4x-1)^2 - 3(4x-1)$

$0 \geq (4x-1)[2(4x-1)-3]$

$0 \geq (4x-1)(8x-2-3)$

$0 \geq (4x-1)(8x-5)$

$\dfrac{1}{4} < x \leq \dfrac{5}{8}$



b) $\sin 2x + \cos x = 0$

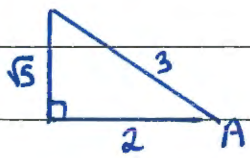
$2\sin x \cos x + \cos x = 0$

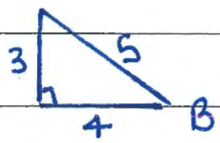
$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ $\quad$ $2\sin x + 1 = 0$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\quad$ $\sin x = -\dfrac{1}{2}$

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

c) 



$B = \tan^{-1}\dfrac{3}{4}$

$\tan B = \dfrac{3}{4}$

$\cos A = \dfrac{2}{3}$ $\quad$ $\sin B = \dfrac{3}{5}$

$\sin A = \dfrac{\sqrt{5}}{3}$ $\quad$ $\cos B = \dfrac{4}{5}$

$= \sin\left[\cos^{-1}\left(\dfrac{2}{3}\right) + \tan^{-1}\left(-\dfrac{3}{4}\right)\right]$

$= \sin\left[\cos^{-1}\dfrac{2}{3} - \tan^{-1}\left(\dfrac{3}{4}\right)\right]$

$\sin(A-B)$

$= \sin A \cos B - \cos A \sin B$

$= \frac{\sqrt{5}}{3} \times \frac{4}{5} - \frac{2}{3} \times \frac{3}{5}$

$= \frac{4\sqrt{5}-6}{15}$

d) $u = x^2$ $\quad\quad v = \cos^{-1}(2x)$

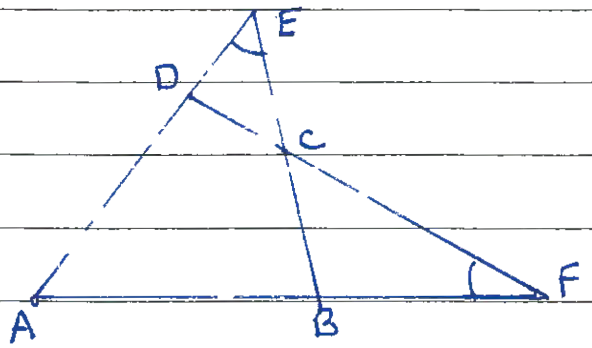
$du = 2x\,dx$ $\quad\quad dv = \frac{-1}{\sqrt{1-(2x)^2}} \times 2$

$\frac{d}{dx}\left(x^2\cos^{-1}(2x)\right)$

$= \cos^{-1}2x \times 2x + x^2 \times \frac{-1}{\sqrt{1-(2x)^2}} \times 2$

$= 2x\cos^{-1}(2x) - \frac{2x^2}{\sqrt{1-4x^2}}$

e)



In ΔDEC

$\angle ADC = \angle DEC + \angle ECD$

exterior angle is equal to the sum of the two opposite interior angles.

In ΔBFC

$\angle ABC = \angle BFC + \angle FCB$ (exterior angle is equal to the sum of the two opposite interior angle)

$\angle DCE = \angle BCF$ (vertically opposite angles equal)

$\therefore \angle ADC = \angle ABC$

e ii) $\angle ADC + \angle ABC = 180°$

(opposite angles of a cyclic quadrilateral are supplementary)

$\therefore \angle ABC = 90°$

$\therefore$ AC is diameter of circle ABCD

Question 12

a) $S_n = 1\times 2^0 + 2\times 2^1 + 3\times 2^2 + \ldots + n\times 2^{n-1} = 1 + (n-1)2^n$

Show true for $n=1$

$L.H.S = 1\times 2^{1-1}$ $\quad$ $R.H.S = 1 + (1-1)2^1$

$= 1$ $\quad$ $= 1$

$\therefore$ true for $S(1)$

Assume true for $n=k$

ie $1\times 2^0 + 2\times 2^1 + 3\times 2^2 + \ldots + k\times 2^{k-1} = 1 + (k-1)2^k$

Prove true for $n=k+1$

ie prove $1\times 2^0 + 2\times 2^1 + 3\times 2^2 + \ldots + k\times 2^{k-1} + (k+1)2^k = 1 + k\times 2^{k+1}$

$L.H.S = 1\times 2^0 + 2\times 2^1 + 3\times 2^2 + \ldots + k\times 2^{k-1} + (k+1)\times 2^k$

$= 1 + (k-1)2^k + (k+1)2^k$

$= 1 + (k-1+k+1)2^k$

$= 1 + 2k\cdot 2^k$

$= 1 + [(k+1)-1]2^{k+1}$

$= RHS$

If $S(k)$ is true then $S(k+1)$ is true, $S(1)$ is true, $S(2)$ is true and then $S(3)$ is true and so on.

$\therefore S(n)$ is true for all positive integers $n \geq 1$

b) $A = \pi r^2$

$\frac{dr}{dt} = 0.1$

$\frac{dA}{dt} = 2\pi$

$r = ?$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$2\pi = 2\pi r \times 0.1$

$0.1r = 1$

$r = 10$

$\therefore$ Radius is 10m when the area is increasing at $2\pi \ m^2/s$

c i) $x^2 = 4ay$

$y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$

At $P(2ap, ap^2)$

$M_{tangent} = \frac{2ap}{2a} = p$

$M_{chord} = \frac{ap^2 - aq^2}{2ap - 2aq}$

$= \frac{a(p-q)(p+q)}{2a(p-q)}$

$= \frac{p+q}{2}$

$M_{tangent} = 4 \times M_{chord}$

$p = 4 \times \frac{p+q}{2}$

$p = 2p + 2q$

$-p = 2q$

$\therefore q = \frac{1}{2}p$ as required c ii)

$$M = \left(\frac{2ap + 2aq}{2}, \frac{ap^2 + aq^2}{2}\right) \quad -①$$

$$q = -\frac{1}{2}p \quad -②$$

② → ①

$$M = \left(\frac{2ap + 2a\left(-\frac{p}{2}\right)}{2}, \frac{ap^2 + a\left(-\frac{p}{2}\right)^2}{2}\right)$$

$$= \left(\frac{2ap - ap}{2}, \frac{ap^2 + \frac{ap^2}{4}}{2}\right)$$

$$= \left(\frac{ap}{2}, \frac{5ap^2}{8}\right)$$

locus of M = $\quad x = \frac{ap}{2} \qquad p = \frac{2x}{a}$

$$y = \frac{5ap^2}{8}$$

$$y = \frac{5a}{8} \times p^2$$

$$y = \frac{5a}{8} \times \left(\frac{2x}{a}\right)^2$$

$$y = \frac{5a}{8} \times \frac{4x^2}{a^2}$$

$$y = \frac{5x^2}{2a}$$

$2ay = 5x^2$ which is the form of a parabola

∴ M moves on a parabola.

d) $u = x+1$ $\qquad x = 2 \implies u = 3$

$du = .$ $\qquad x = 5 \implies u = 6$

$$\int_2^5 \frac{x+2}{(x+1)^2}\,dx = \int_3^6 \frac{u+1}{u^2}\,du$$

$$= \int_3^6 \frac{1}{u} + \frac{1}{u^2}\,du$$

$$= \left[\ln u - \frac{1}{u}\right]_3^6$$

$$= \left(\ln 6 - \frac{1}{6}\right) - \left(\ln 3 - \frac{1}{3}\right)$$

$$= (\ln 6 - \ln 3) - \left(\frac{1}{6} - \frac{1}{3}\right)$$

$$= \ln 2 + \frac{1}{6}$$

e) $y = \cos^{-1} x$

$x = \cos y$

$$V = \pi \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \cos^2 y\,dy$$

$$= \frac{\pi}{2} \int_{\pi/12}^{\frac{\pi}{4}} (1 + \cos 2y)\,dy$$

$$= \frac{\pi}{2}\left[y + \frac{1}{2}\sin 2y\right]_{\frac{\pi}{12}}^{\frac{\pi}{4}}$$

$$= \frac{\pi}{2}\left[\left(\frac{\pi}{4} - \frac{\pi}{12}\right) - \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right)\right]$$

$$= \frac{\pi}{2}\left(\frac{\pi}{6} + \frac{1}{4}\right) \text{ units}^3$$

Question 13

a i) $N = P + 100e^{kt}$

$$\frac{dN}{dt} = k(100e^{kt})$$

$$= k(N - P)$$

ii) $t = 0$, $N = 600$

$$600 = P + 100e^{k \times 0}$$

$$600 = P + 100$$

$$P = 500$$

$$\frac{dN}{dt} = 12$$

$$= k(100e^{kt})$$

$$12 = 100k$$

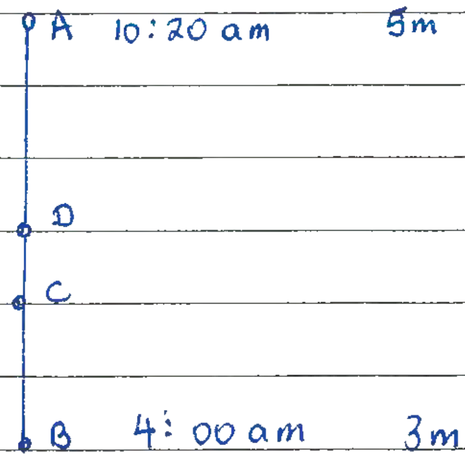
$$k = 0.12$$

b) i) SHM $\quad y = a - b\cos nt$

a - centre of motion

b - amplitude

$\frac{2\pi}{n}$ - period



By inspection $a = 4 \quad b = 1$

Time for one cycle is $2 \times 6\frac{1}{3}$ hours

$$\frac{2\pi}{n} = 12\frac{2}{3}$$

$$n = \frac{3\pi}{19}$$

$\therefore$ Water depth is given by $y = 4 - \cos\frac{3\pi t}{19}$

b ii) $y = 4.5$ $\quad 4 - \cos\frac{3\pi t}{19} = 4.5$

$t = ?$

$$\cos\frac{3\pi t}{19} = -\frac{1}{2}$$

$$\frac{3\pi t}{19} = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$t = \frac{38}{9} \text{ hours } (4\text{h } 13\text{ min})$$

$$t = \frac{76}{9} \text{ hours } (8\text{ h } 26\text{ min})$$

∴ Boat can be in the harbour between 8:13am and 12:26pm

c) i) $v = \frac{dx}{dt} = \frac{16 - x^2}{x}$

$$\frac{dt}{dx} = \frac{x}{16 - x^2}$$

$$\frac{dt}{dx} = -\frac{1}{2} \times \frac{-2x}{16 - x^2}$$

$$t = -\frac{1}{2}\ln(16 - x^2) + c$$

When $t = 0$ $\quad x = 1$

$$0 = -\frac{1}{2}\ln 15 + c$$

$$c = \frac{1}{2}\ln 15$$

$$t = -\frac{1}{2}\ln\left(\frac{16 - x^2}{15}\right)$$

$$-2t = \ln\left(\frac{16 - x^2}{15}\right)$$

$$e^{-2t} = \frac{16 - x^2}{15}$$

$$15e^{-2t} = 16 - x^2$$

$$x^2 = 16 - 15e^{-2t} \qquad x > 0 \text{ for } t = 0$$

$$x = \sqrt{16 - 15e^{-2t}}$$

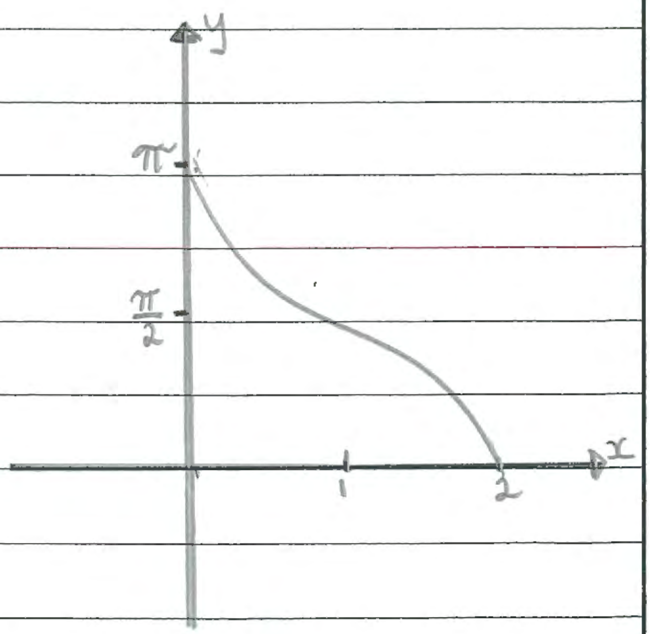
c ii) As $t \to \infty$

$e^{-2t} \to 0$

$\therefore x$ approaches $\sqrt{16} = 4$

Particle moves right towards a limiting position 4m right of 0.

d) $f(x) = \sin^{-1}(1-x) + \frac{\pi}{2}$



Domain $-1 \leq 1 - x \leq 1$

$-1 \leq x - 1 \leq 1$

$0 \leq x \leq 2$

Range $-\frac{\pi}{2} \leq \sin^{-1}(1-x) \leq \frac{\pi}{2}$

$0 \leq \sin^{-1}(1-x) + \frac{\pi}{2} \leq \pi$

$0 \leq f(x) \leq \pi$

Question 4

a i) The maximum height occurs when $\dot{y}=0$

$\dot{y} = 70\sin\theta - 9.8t$

$0 = 70\sin\theta - 9.8t$

$t = \dfrac{70\sin\theta}{9.8}$

$\therefore\ y = \dfrac{70 \times 70\sin^2\theta}{9.8} - \dfrac{4.9 \times 70 \times 70 \times \sin^2\theta}{9.8 \times 9.8}$

$= 500\sin^2\theta - 250\sin^2\theta$

$= 250\sin^2\theta$

a ii) $t = \dfrac{70\sin\theta}{9.8}$

$x = 70 \times \dfrac{70\sin\theta}{9.8} \times \cos\theta$

$x = 500\cos\theta\sin\theta$

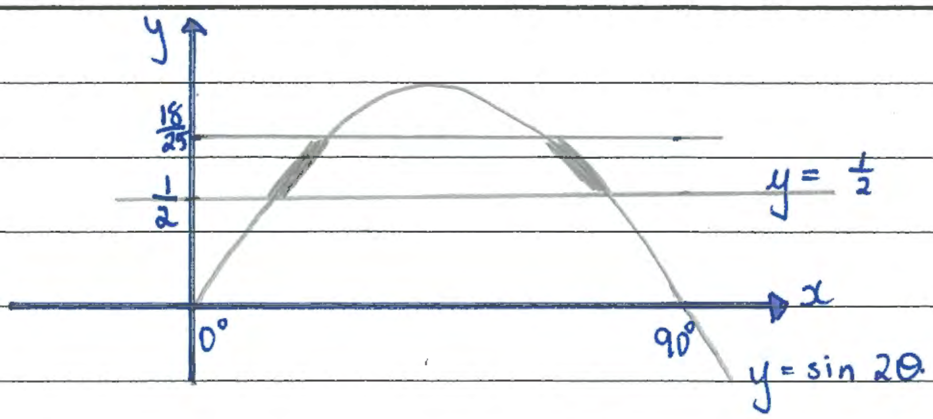
$= 250(2\sin\theta\cos\theta)$

$= 250\sin 2\theta$

$\therefore$ the horizontal distance from O is $250\sin 2\theta$ m

a iii) We want $125 \leq 250\sin 2\theta \leq 180$

$\frac{1}{2} \leq \sin 2\theta \leq \frac{18}{25}$



$$\frac{1}{2} \leq \sin 2\theta \leq \frac{18}{25}$$

$30° \leq 2\theta \leq 46°$ $134° \leq 2\theta \leq 150°$

$15° \leq \theta \leq 23°$ $67° \leq \theta \leq 75°$

for $y \geq 150°$ and $0° \leq \theta \leq 90°$

$$250 \sin^2 \theta \geq 150°$$

$$\sin^2 \theta \geq \frac{150}{250}$$

$$\sin^2 \theta \geq 0.6$$

$$\sin \theta \geq 0.774$$

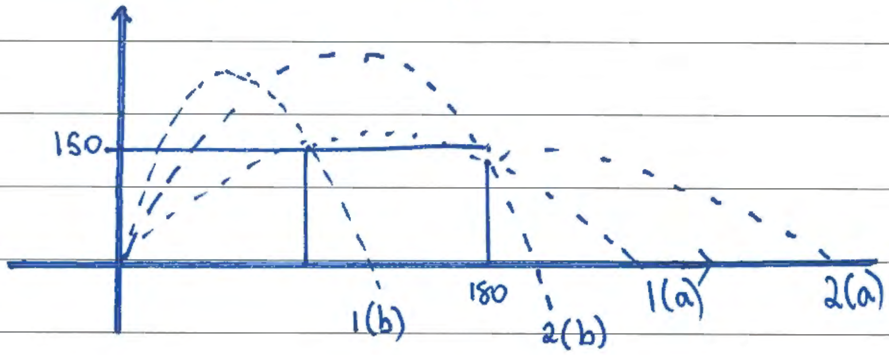
$$\theta \geq 50.76$$

$15° \leq \theta \leq 23°$ $67° \leq \theta \leq 75°$

To satisfy both conditions

$$67° \leq \theta \leq 75°$$

Other method for 14 a iii)



$y = x\tan\theta - \frac{x^2}{1000} - \frac{x^2}{1000}\tan^2\theta$

To pass through (125, 150)

$150 = 125\tan\theta - \frac{125^2}{1000} - \frac{125^2}{1000}\tan^2\theta$

$5\tan^2\theta - 40\tan\theta + 53 = 0$

$\tan\theta = 6.324$ or 1.676 (by quadratic function)

$\theta = 81°$ or $59°11$ (flight shown above)

answer 1(b) answer 1(a)

To pass through (180, 150)

$150 = 180\tan\theta - \frac{180^2}{1000} - \frac{180^2}{1000}\tan^2\theta$

$\therefore 162\tan^2\theta - 900\tan\theta + 912 = 0$

$\therefore \tan\theta = 4.22$ or 1.33

$\theta = 76°41'$ $\theta = 53°4'$

answer 2(b) answer 2(a)

To denote in the required area, the rocket needs to be launched between 1(a) and 2(b) ie between 59°11' and 76°41' b i) $1 + x + x^2 + x^3 + \dots + x^n$

$r = 1$ $\quad S_n = \dfrac{1(x^{n+1} - 1)}{x - 1}$

$n+1$ terms

$S_n = \dfrac{x^{n+1} - 1}{x - 1}$

b ii) Differentiating both sides

$$1 + x + x^2 + x^3 + \dots + x^n = \frac{x^{n+1} - 1}{x - 1}$$

$$1 + 2x + 3x + \dots + nx^{n-1} = \frac{(x-1)(n+1)x^n - (x^{n+1} - 1)\cdot 1}{(x-1)^2}$$

Sub $x = 2$ into both sides

$$1 + 2\times 2 + 3\times 2 + \dots + n\times 2^{n-1} = \frac{(2-1)(n+1)2^n - (2^{n+1} - 1)\cdot 1}{(2-1)^2}$$

$$= (n+1)2^n - 2^{n+1} + 1$$

$$= n2^n + 2^n - 2^{n+1} + 1$$

$$= n2^n + 2^n - 2\cdot 2^n + 1$$

$$= 2^n(n + 1 - 2) + 1$$

$$= 2^n(n-1) + 1$$

as required.

c i)

$\sin\alpha = \frac{PG}{u}$ $\cos\alpha = \frac{BG}{u}$

$PG = u\sin\alpha$ $BG = u\cos\alpha$

In ΔGBA

$AG^2 = BG^2 + 1^2 - 2 \times BG\cos 60^\circ$

$= BG^2 + 1^2 - 2 \times BG \times \frac{1}{2}$

$= BG^2 + 1 - BG$

$= (u\cos\alpha)^2 + 1 - (u\cos\alpha)$

In ΔAPG

$r^2 = PG^2 + AG^2$

$= (u\sin\alpha)^2 + (u^2\cos^2\alpha + 1 - u\cos\alpha)$

$= u^2\sin^2\alpha + u^2\cos^2\alpha + 1 - u\cos\alpha$

$= u^2(\sin^2\alpha + \cos^2\alpha) + 1 - u\cos\alpha$

$r^2 = u^2 + 1 - u\cos\alpha$

$r = \sqrt{1 + u^2 - u\cos\alpha}$

c ii,

$$\frac{du}{dt} = 360 \qquad t = 5\text{ min or } \frac{1}{12}\text{ hour}$$

$$u = 360 \times \frac{1}{12} = 30\text{ km}$$

$$r = \sqrt{1+u^2-u\cos\alpha}$$

$$= (1+u^2-u\cos\alpha)^{\frac{1}{2}}$$

$$\frac{dr}{du} = \frac{1}{2}(1+u^2-u\cos\alpha)^{-\frac{1}{2}}(2u-\cos u)$$

$$= \frac{2u-\cos\alpha}{2\sqrt{1+u^2-u\cos\alpha}}$$

As $u = 30$

$$\frac{dr}{du} = \frac{2\times 30-\cos\alpha}{2\sqrt{1+30^2-30\cos\alpha}}$$

$$= \frac{60-\cos\alpha}{2\sqrt{901-30\cos\alpha}}$$

$$\frac{dr}{dt} = \frac{dr}{du} \times \frac{du}{dt}$$

$$= \frac{60-\cos\alpha}{2\sqrt{901-30\cos\alpha}} \times 360$$

$$= 180\left(\frac{60-\cos\alpha}{\sqrt{901-30\cos\alpha}}\right)\text{ km/h}$$

TRIAL HSC 2019 EXTENSION 1 – Markers comments

Question 11:

a) $x \neq \frac{1}{4}$ needs

b) Please don't divide by $cosx$ as you lose half the answers and remember the correct quadrants in RADIANS.

c) $\tan^{-1}(\frac{-3}{4})$ = 4th quadrant

d) Learn the rule to differentiate inverse cosine

e) Learn the correct reasons and state which triangle etc you are referring to. Most students had very poor setting out and reasoning.

Question 12:

a) Please learn how to correctly set out mathematical induction. Use LHS = ….. = RHS method working methodically down the page. Also, don't forget to add that k is a positive integer.

b) Some students did not realise that you need to link the area of a circle to obtain $\frac{dA}{dr} = 2\pi r$ and substitute in the given information to find the value of r.

c) (i) Show means do not skip steps. You must derive the gradient of the tangent at P and the gradient of the chord PQ first for one mark.

(ii) Poorly done question. You must always eliminate the parameter and show the locus in Cartesian form.

d) Don't forget to change the bounds to be in terms of u. Silly mistakes when integrating $\frac{1}{u^2}$ lead to carry errors.

e) Several students thought that $\frac{\pi}{12}$ is a larger fraction than $\frac{\pi}{4}$ and put the bounds in the wrong spot. Many stated that the integral of $cos2y$ is $-\frac{1}{2}sin2y$. You should know your basic integrals and if necessary, refer to the reference sheet to check and avoid silly mistakes with the signs.

Question 13:

aii) about half of you didn't solve this correctly, "initially" means t=0 so no log involved.

bi) small sketch or mention of amplitude needed to be made, not just assumption. n needed to be calculated with formula, centre of motion is 4. This with some explanation, achieved 2 marks.

ci) one mark deducted if in final step no mention of initial condition.

d) If not sure of shape of graph or where to start, make substitutions.